

Chapter 3 Expressions and Equations

Four Square

1–7. Sample answers are given.

1.

<p>Definition</p> <p>An algebraic expression is in <i>simplest form</i> when it has:</p> <ol style="list-style-type: none"> 1. no like terms and 2. no parentheses. 	<p>Words</p> <p>To write an algebraic expression in simplest form:</p> <p>Step 1: Rewrite as a sum.</p> <p>Step 2: Use the Distributive Property on parentheses, if necessary.</p> <p>Step 3: Rearrange terms.</p> <p>Step 4: Combine like terms.</p>
<p>Simplest form</p>	
<p>Example</p> $5x^2 + 6x - 3x^2 + 8 - x$ $= 5x^2 + 6x + (-3x^2) + 8 + (-1x)$ $= 5x^2 + (-3x^2) + 6x + (-1x) + 8$ $= [5 + (-3)]x^2 + [6 + (-1)]x + 8$ $= 2x^2 + 5x + 8$	<p>Example</p> $9 - 3\left(\frac{2}{3}m - \frac{1}{3}\right) + 3m$ $= 9 + (-3)\left(\frac{2}{3}m + \left(-\frac{1}{3}\right)\right) + 3m$ $= 9 + (-3)\left(\frac{2}{3}m\right) + (-3)\left(-\frac{1}{3}\right) + 3m$ $= 9 + (-2m) + 1 + 3m$ $= (-2m) + 3m + 9 + 1$ $= (-2 + 3)m + (9 + 1)$ $= m + 10$

2.

<p>Definition</p> <p>An algebraic expression in which the exponent of the variable is 1</p>	<p>Examples</p> $-7x, 2x + 3, 8 - \frac{1}{4}x$ <p>Non-examples:</p> $x^3, -5x^2 + x, x^7 - 9$
<p>Linear expression</p>	
<p>Example</p> <p>Adding linear expressions:</p> $(7 - w) + 3(-2w + 4)$ $= 7 + (-1w) + 3(-2w) + 3(4)$ $= 7 + (-1w) + (-6w) + 12$ $= (-1w) + (-6w) + 7 + 12$ $= -7w + 19$	<p>Example</p> <p>Subtracting linear expressions:</p> $(4y + 7) - (y - 8)$ $4y + 7 \Rightarrow 4y + 7$ $-(y - 8) \Rightarrow +(-1y) + 8$ $\underline{\hspace{1.5cm} 3y + 15}$

3.

<p>Words</p> <p>Write the expression as a product of factors. You can use the Distributive Property.</p>	<p>Example</p> <p>Factor $12a - 30$ using the GCF.</p> $12a = 2 \cdot 2 \cdot 3 \cdot a$ $30 = 2 \cdot 3 \cdot 5$ $\text{GCF} = 2 \cdot 3 = 6$ $12a - 30 = 6(2a) - 6(5)$ $= 6(2a - 5)$
<p>Factoring expressions</p>	
<p>Example</p> <p>Factor $\frac{1}{4}$ out of $\frac{1}{4}r + \frac{3}{4}$.</p> $\frac{1}{4}r = \frac{1}{4} \cdot r$ $\frac{3}{4} = \frac{1}{4} \cdot 3$ $\frac{1}{4}r + \frac{3}{4} = \frac{1}{4} \cdot r + \frac{1}{4} \cdot 3$ $= \frac{1}{4}(r + 3)$	<p>Example</p> <p>Factor -7 out of $-21p + 28$.</p> $-21p = -7 \cdot 3p$ $28 = -7 \cdot (-4)$ $-21p + 28 = -7(3p) + (-7)(-4)$ $= -7(3p - 4)$

4.

<p>Words</p> <p>Two equations are <i>equivalent equations</i> if they have the same solutions. You can use the Addition, Subtraction, Multiplication, and Division Properties of Equality to write equivalent equations.</p>	<p>Algebra</p> $a = b \text{ and } a + c = b + c$ $a = b \text{ and } a - c = b - c$ $a = b \text{ and } a \cdot c = b \cdot c$ $a = b \text{ and } \frac{a}{c} = \frac{b}{c}, c \neq 0$
<p>Equivalent equations</p>	
<p>Examples</p> $x - 7 = 2 \text{ and } x - 7 + 7 = 2 + 7$ $2d + 5 = -7 \text{ and } 2d + 5 - 5 = -7 - 5$ $24 = \frac{y}{-4} \text{ and } -4 \cdot 24 = -4 \cdot \frac{y}{-4}$ $3c = -12 \text{ and } \frac{3c}{3} = \frac{-12}{3}$	<p>Non-Examples</p> $x + 7 = 2 \text{ and } x + 7 - 7 = 2 + 7$ $3c = -4 \text{ and } \frac{3c}{3} = 3 \cdot (-4)$ $7 = m + 3 \text{ and } 7 - 7 = m + 3 - 3$ $3x + 7 = 3 \text{ and } 3x = 7 + 3$

Chapter 3 (continued)

5.

<p>Words</p> <p>To undo addition, use the Subtraction Property of Equality: subtracting the same number from each side of an equation produces an equivalent equation. To undo subtraction, use the Addition Property of Equality: adding the same number to each side of an equation produces an equivalent equation.</p>	<p>Algebra</p> <p>If $a + b = c$, then $a + b - b = c - b$.</p> <p>If $a - b = c$, then $a - b + b = c + b$.</p>
<p>Solving equations using addition or subtraction</p>	
<p>Example</p> $x - 4 = 12$ $\begin{array}{r} +4 \quad +4 \\ x - 4 = 12 \\ \hline x = 16 \end{array}$ <p style="text-align: right;">Check</p> $x - 4 = 12$ $16 - 4 \stackrel{?}{=} 12$ $12 = 12$ <p style="text-align: right;">✓</p>	<p>Example</p> $x + 4.1 = 12$ $\begin{array}{r} -4.1 \quad -4.1 \\ x + 4.1 = 12 \\ \hline x = 7.9 \end{array}$ <p style="text-align: right;">Check</p> $x + 4.1 = 12$ $7.9 + 4.1 \stackrel{?}{=} 12$ $12 = 12$ <p style="text-align: right;">✓</p>

6.

<p>Words</p> <p>To undo multiplication, use the Division Property of Equality: dividing each side of an equation by the same number produces an equivalent equation. To undo division, use the Multiplication Property of Equality: multiplying each side of an equation by the same number produces an equivalent equation.</p>	<p>Algebra</p> <p>If $ab = c$, then $\frac{ab}{b} = \frac{c}{b}$, $b \neq 0$.</p> <p>If $\frac{a}{b} = c$, then $b \cdot \frac{a}{b} = b \cdot c$, $b \neq 0$.</p>
<p>Solving equations using multiplication or division</p>	
<p>Example</p> $-2x = 3$ $\begin{array}{r} -2x = 3 \\ \hline -2 \quad -2 \\ -2x = 3 \\ \hline x = -\frac{3}{2}, \text{ or } -1\frac{1}{2} \end{array}$ <p style="text-align: right;">Check</p> $-2x = 3$ $-2\left(-\frac{3}{2}\right) \stackrel{?}{=} 3$ $3 = 3$ <p style="text-align: right;">✓</p>	<p>Example</p> $\frac{x}{2} = -3$ $2 \cdot \frac{x}{2} = 2 \cdot (-3)$ $x = -6$ <p style="text-align: right;">Check</p> $\frac{x}{2} = -3$ $\frac{-6}{2} \stackrel{?}{=} -3$ $-3 = -3$ <p style="text-align: right;">✓</p>

7.

<p>Words</p> <p>Undo the operations in the reverse order of the order of operations:</p> <ol style="list-style-type: none"> Undo addition or subtraction. Undo multiplication or division. <p>After solving for the variable, check your solution.</p>	<p>Example</p> $3x + 4 = 1$ $\begin{array}{r} -4 \quad -4 \\ 3x + 4 = 1 \\ \hline 3x = -3 \\ \hline \frac{3x}{3} = \frac{-3}{3} \\ x = -1 \end{array}$ <p style="text-align: right;">Check</p> $3x + 4 = 1$ $3(-1) + 4 \stackrel{?}{=} 1$ $-3 + 4 \stackrel{?}{=} 1$ $1 = 1$ <p style="text-align: right;">✓</p>
<p>Solving two-step equations</p>	
<p>Example</p> $\frac{3}{7} - \frac{a}{3} = -\frac{4}{7}$ $\begin{array}{r} -\frac{3}{7} \quad -\frac{3}{7} \\ \frac{3}{7} - \frac{a}{3} = -\frac{4}{7} \\ \hline -\frac{a}{3} = -1 \\ -3 \cdot \left(-\frac{a}{3}\right) = -3 \cdot (-1) \\ a = 3 \end{array}$ <p style="text-align: right;">Check</p> $\frac{3}{7} - \frac{a}{3} = -\frac{4}{7}$ $\frac{3}{7} - \frac{3}{3} \stackrel{?}{=} -\frac{4}{7}$ $\frac{3}{7} + (-1) \stackrel{?}{=} -\frac{4}{7}$ $\frac{3}{7} + \left(-\frac{7}{7}\right) \stackrel{?}{=} -\frac{4}{7}$ $-\frac{4}{7} = -\frac{4}{7}$ <p style="text-align: right;">✓</p>	<p>Example</p> $4(x - 2) = 12$ $4x + (-8) = 12$ $\begin{array}{r} +8 \quad +8 \\ 4x + (-8) = 12 \\ \hline 4x = 20 \\ \hline \frac{4x}{4} = \frac{20}{4} \\ x = 5 \end{array}$ <p style="text-align: right;">Check</p> $4(x - 2) = 12$ $4(5 - 2) \stackrel{?}{=} 12$ $4(3) \stackrel{?}{=} 12$ $12 = 12$ <p style="text-align: right;">✓</p>